Numerical Modeling of Baseball Pitching

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Abstract. In this study, the forces acting on a baseball pitch (gravity, air drag, and Magnus force) are considered. A numerical procedure is developed using Newton's 2nd Law on four different pitches; a fastball, curveball, slider, and screwball. Using a 4th order Runge Kutta numerical integration technique, trajectories are obtained for the different situations. The results agree very well with what one would intuitively expect and demonstrate that simple physical models can often be very effective.

Keywords: baseball, pitching, fastball, curveball, slider, screwball

1. Introduction

Baseball is the favorite summer pastime in many countries, but it is also a sport embedded with deep principles in physics. In baseball, a pitcher throws a small spherical object towards a batter who attempts to hit this ball with a block of wood. The ball is subject to various forces as it travels through the air and it is governed by Newton's laws of motion. Then a violent and short collision takes place between the bat and the ball with huge amounts of energy dissipation depending on the impact location. Finally, the ball flies off of the bat and through the air.

This paper focuses on modeling four different common pitches in baseball: the fastball, curveball, slider (like a curveball but it drops more), and screwball (like a curveball but spins and curves in opposite direction). A simple theoretical model based on Newton's 2nd Law was used.

2. Theory

As a baseball travels through the air, it is acted on by three dominant forces: *gravity, air drag*, and the *Magnus force*. Gravity is the most straightforward of the three forces; if we define the positive z direction to be upwards, then the force due to gravity on the ball at any point on its trajectory is simply

$$\mathbf{F_g} = -mg\,\hat{\mathbf{z}} \tag{1}$$

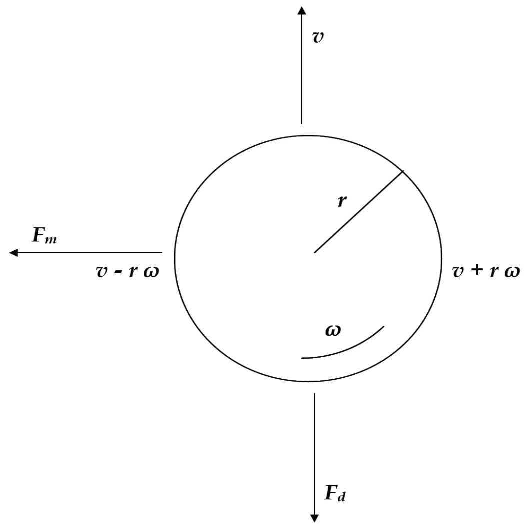

Figure 1. A pictorial representation of the origin of the Magnus force. The diagram shows a view of the ball from above (so gravity is acting into the page). Due to the rotation, the velocity on the right side of the baseball is greater than the velocity on the left side, resulting in a net force to the left.

In this equation, m is the mass of the ball and g is the acceleration due to gravity.

One may think that the drag force on a baseball can be expressed by simply using Prandtl's relationship, and to first order, that is a good approximation. Wind tunnel measurements, however, have shown that the so-called *drag coefficient* is a strong function of velocity and surface type for baseballs. As the velocity of a baseball increases and the air flow around the baseball switches from laminar to turbulent, the drag coefficient drops and the baseball slips more easily through the air. This transition regime occurs at a very high velocity for *smooth* baseballs, but it turns out that it is at a much lower velocity for rough baseballs. This counterintuitive observation states that a rough baseball will experience less drag than a smooth one.

An empirical formula giving the drag force on a baseball of velocity v has been developed by Giordano (1997).

$$\mathbf{F_d} = mf(v)v\,\mathbf{v} \tag{2}$$

where v is the translational velocity of the baseball and the function $f(v)$ is given by

$$f(v) = 0.0039 + \frac{0.0058}{1 + \exp\left[\left(v - v_d\right)/\Delta\right]} \tag{3}$$

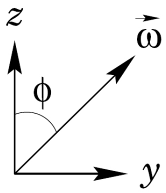

Figure 2. Definition of the angle ϕ for expressing the angular velocity vector

In Equation 3, the quantities v_d and Δ are empirical constants and have values of 35 m/s and 5 m/s respectively.

The drag force explains why the baseball slows down as it approaches home plate, but it does not explain why certain pitches *curve* horizontally during flight. This curvature occurs due to a phenomenon called the *Magnus force* which originates due to the spinning of the baseball. Assume that the ball is spinning about an axis perpendicular to its direction of motion (this is almost always the case). The velocity at any point on the ball will have two components because the center of mass of the ball is moving *and* the entire ball is spinning. This causes the velocity vector to be different at different points on the ball; in other words, some parts of the ball are moving faster than others. This is illustrated in Figure 1 where, in the diagram, the ball is moving upwards and rotating counterclockwise. In this case, the velocity is greatest on the rightmost edge and lowest on the leftmost edge. Since drag force increases with velocity and we have a velocity gradient throughout the ball, this means that the drag force will also have a horizontal component. The velocity is highest on the rightmost edge of the ball, so there will be a net drag force pointing to the left. This is the so-called *Magnus force*, $\mathbf{F_m}$, which is the dominant spin-related force acting on a baseball. Recall that the Magnus force is really just a special case of the aerodynamic drag force.

It turns out that the Magnus force can be written as the following cross product,

$$\mathbf{F_m} = S(v)\, \boldsymbol{\omega} \times \mathbf{v} \tag{4}$$

where $\boldsymbol{\omega}$ is the angular velocity vector and $S(v)$ is some unknown function of velocity. Adair (1990) suggests that for a typical baseball pitch, S can be somewhat well approximated by

$$S = \left(4.1 \times 10^{-4}\right) m \tag{5}$$

Table I. The parameters corresponding to each modeled baseball pitch. Note that v_0 is the initial velocity, ω is the angular velocity, θ is the elevation angle, and ϕ is the angular orientation (see Figure 2 for more information on ϕ)

rpm should be converted to rad/s: 1800*2*pi/60

Pitch	v_0 (mph)	ω (rpm)	$\theta(^\circ)$	$\phi(^\circ)$
Fastball	95	1800	1	225
Curveball	85	1800	1	45
Slider	85	1800	1	0
Screwball	85	1800	1	135

convert to meters per sec

convert deg to rad

3. Computational Model

Allow us to choose a coordinate system where x represents displacement from the pitcher to the hitter, y represents movement from side to side (the ball would move to the pitcher's left if it moves in the +y direction), and z represents vertical motion. The three dominant forces acting on a baseball are the force of gravity $\mathbf{F_g}$ (Equation 1), the drag force $\mathbf{F_d}$ (Equation 2), and the Magnus force $\mathbf{F_m}$ (Equation 4). We define the angular velocity vector as $\boldsymbol{\omega} = \omega\,(0, \sin\phi, \cos\phi)$ where ϕ represents the angle between the z-axis and the baseball's angular velocity vector (see Figure 2). In addition, we give the velocity vector a general definition of $\mathbf{v} = (v_x, v_y, v_z)$. This allows to expand Equation 4 using the definition of the cross product.

$$\begin{aligned} \mathbf{F_m} &= S(v)\,\boldsymbol{\omega} \times \mathbf{v} \\ &= mB\omega\left[(v_z \sin\phi - v_y \cos\phi)\,\hat{\mathbf{x}} + v_x \cos\phi\,\hat{\mathbf{y}} - v_x \sin\phi\,\hat{\mathbf{z}}\right] \end{aligned} \tag{6}$$

Recalling that the velocity is the time derivative of displacement, acceleration is the time derivative of velocity, and $\mathbf{F} = m\mathbf{a}$, the equations of motion of a baseball thrown by a pitcher can be described using the following six coupled ordinary differential equations. Note that Equations 1, 2, and 4 have been used to obtain these equations.

$$\frac{dx}{dt} = v_x \tag{7}$$

$$\frac{dy}{dt} = v_y \tag{8}$$

$$\frac{dz}{dt} = v_z \tag{9}$$

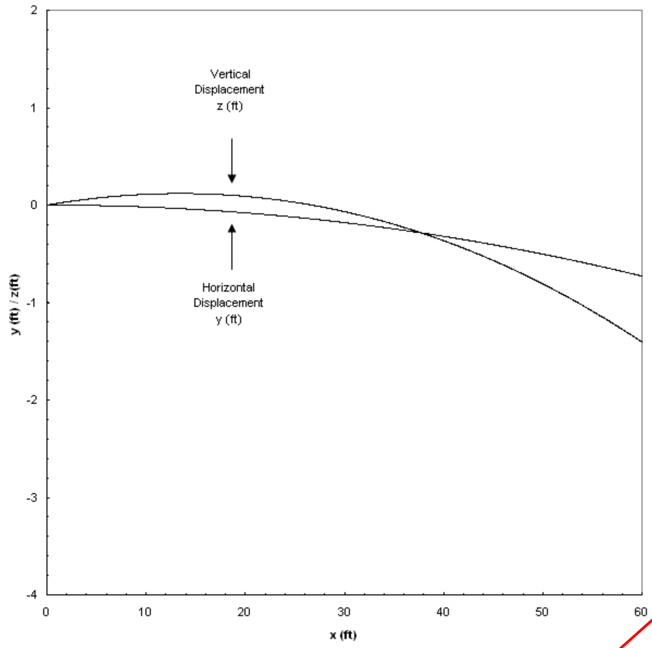

Figure 3. Numerical solution for the trajectory of a fastball delivered [illegible] right-handed pitcher. The fastball parameters are given in Table I.

this should be a minus sign to agree with equation 6

$$\frac{dv_x}{dt} = -f(v)\,vv_x + B\omega\,(v_z\sin\phi - v_y\cos\phi) \tag{10}$$

$$\frac{dv_y}{dt} = -f(v)\,vv_y + B\omega v_x\cos\phi \tag{11}$$

$$\frac{dv_z}{dt} = -f(v)\,vv_z + B\omega v_x\sin\phi \tag{12}$$

there needs to be a gravity term here: -9.8

The boundary conditions applied within the model were:

$$x\,(t=0) = 0 \tag{13}$$

$$y\,(t=0) = 0 \tag{14}$$

$$z\,(t=0) = 0 \tag{15}$$

$$v_x\,(t=0) = v_0\cos\theta \tag{16}$$

$$v_y\,(t=0) = 0 \tag{17}$$

$$v_z\,(t=0) = v_0\sin\theta \tag{18}$$

In these equations, v_0 is the initial speed of the pitch and θ is the angle of elevation.

The above differential equations were numerically solved using a 4th order Runge Kutta technique with a fixed stepsize $t = 10^{-4}\,\mathrm{s}$

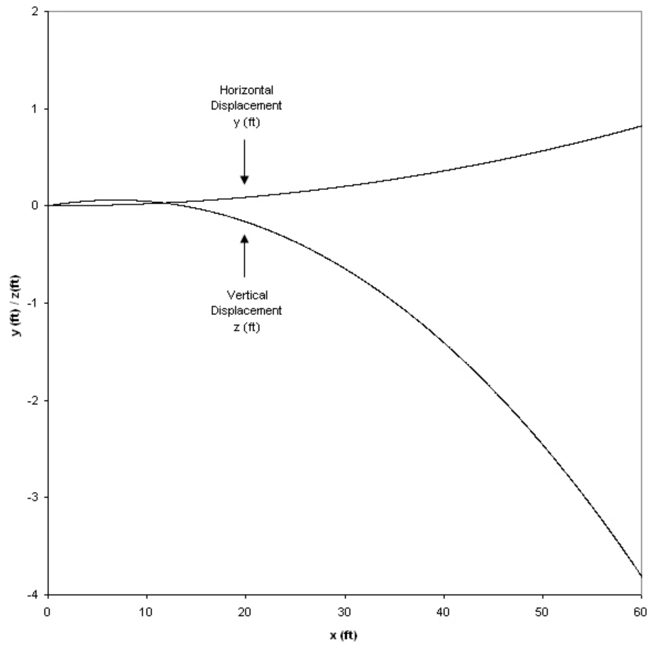

Figure 4. Numerical solution for the trajectory of a curveball delivered by a right-handed pitcher. The curveball parameters are given in Table I.

(chosen to optimize efficiency and accuray). The precise details of the implementation have been omitted since 4th order Runge Kutta is a common, perhaps the most common, method for solving a system of coupled differential equations.

4. Results

Four of the most common pitches in baseball were modeled in this study: a fastball, curveball, slider, and screwball. The values for the various parameters for each pitch are given in Table I. Graphs representing vertical and horizontal position versus displacement from the pitcher's hand have been plotted for each case in Figures 3 (fastball), 4 (curveball), 5 (slider), and 6 (screwball).

The simulation of a fastball (Figure 3) shows that the vertical position only decreased by only around 1.5 feet during the trajectory and the horizontal position changed only by around 0.5 feet. This is expected, however, since the fastball has a lot of backspin and the Magnus force would tend to counteract its downward force due to gravity. Pitchers will often throw a *rising fastball*, one with a lot of spin, and

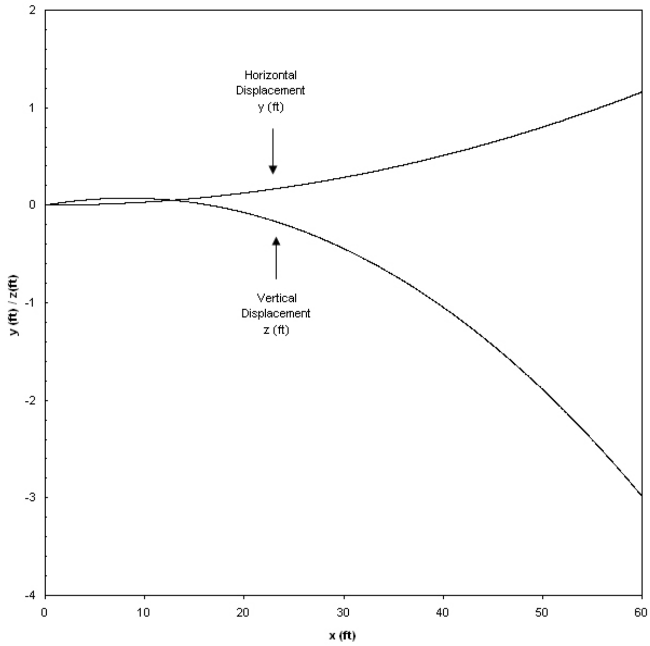

Figure 5. Numerical solution for the trajectory of a slider delivered by a right-handed pitcher. The slider parameters are given in Table I.

hitters often claim that the ball actually rises as it approaches them. This is an optical illusion, but it is interesting that the baseball does not drop more as it approaches the plate. The fastball required 0.46 s to reach the plate and had a final velocity of 86 mph.

The curveball and slider (Figures 4 and 5) exhibited very similar features. Due to the Magnus force they both curved to the left (if thrown by a right-handed pitcher) from the pitcher's point of view, by similar amounts (around a .33 ft) and they both dropped significantly during flight. As expected by the ball's rotation, the curveball was lower than the slider by around a 0.25 ft by the time it reached the hitter, making it a very difficult pitch to hit. The entire flight for both the curveball and slider was only 0.52 s in duration and each pitch was moving at 76 mph when it reached the plate.

It is clear from experience in hitting that it is more difficult to hit a pitch that curves towards you than one that curves away from you. That is the idea behind the final modeled pitch; a screwball is virtually identical to a curveball except for the fact that it spins in the *opposite direction*. This causes it to curve to the right (as viewed and thrown by a right handed pitcher) by around the same amount as a curveball.

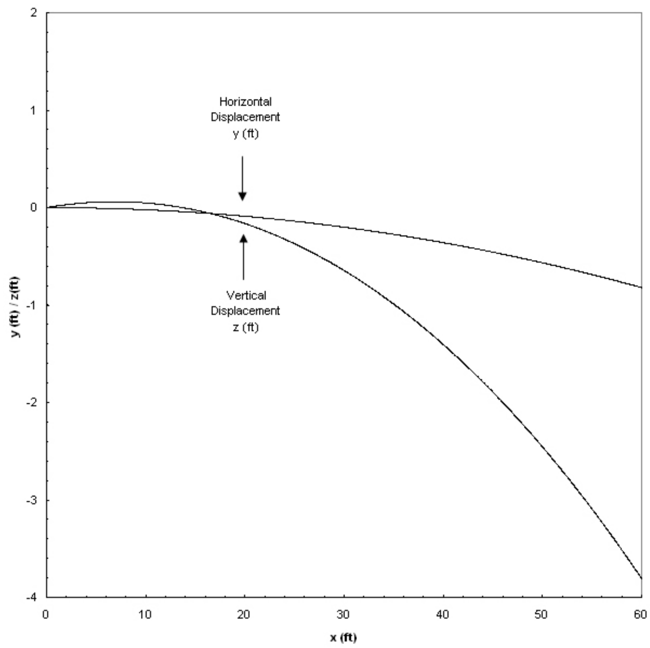

Figure 6. Numerical solution for the trajectory of a screwball delivered by a right-handed pitcher. The screwball parameters are given in Table I.

Like the curveball and slider, the screwball reached home plate after 0.52 s and was traveling at 76 mph.

Note that the z component of each pitch increases slightly before decreasing. This is due to the 1° elevation angle on all pitches; there is a small but non-zero z component of the velocity which causes the ball to slightly increase before gravity dominates the vertical motion.

5. Discussion

The key result of the study is that baseball pitches can be modeled extremely well using only basic physics principles and a simple numerical integrator. We have shown that fastballs do not dip or move a lot but they depend on their speed for their success; on the contrary, we have shown that curveballs, sliders, and screwballs are good pitches because of their movement and not their speed.

Possible improvements on the model would be to obtain better empirical estimates for $f(v)$ (Equation 3) and B (Equation 5). A theoretical calculation of the drag force based on the equations of fluid flow would also be useful to compare with the empirical observations.

Future projects include extending the model to various limiting cases; practical scenarios could be examined, such as the case where the angular velocity of the ball becomes *very* high (this can be achieved through pitching machines) or when the mass becomes very heavy (a waterlogged baseball).

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