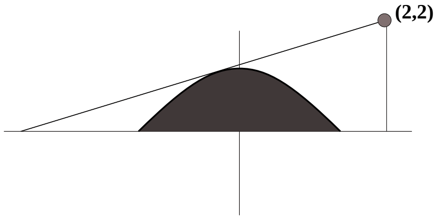
1. Use the definition of derivative and the limit laws to find the derivative of the function $f(x) = 3x^2 - 5x + 9$. Write with complete detail.

2. The figure shows a lamp located 2 units to the right of the y-axis and 2 units above the x-axis. How far along the x-axis does the shadow of the parabolic region $y \le 1 - \frac{1}{2}x^2$ extend?



3. Evaluate the following limits. If you use L'Hôpital's rule, state why it is applicable. Also try to evaluate the limits without appealing to L'Hôpital's rule.

   (a)
   $$\lim_{\theta \to 0} \frac{\sin\theta - \sin\theta\cos\theta}{\theta^2}$$

   (b)
   $$\lim_{\theta \to 0} \frac{\sin^2 3\theta}{5\theta^3 - 5\theta^2}$$

   (c)
   $$\lim_{x \to \infty} \frac{\ln\ln x}{\sqrt{x}}$$

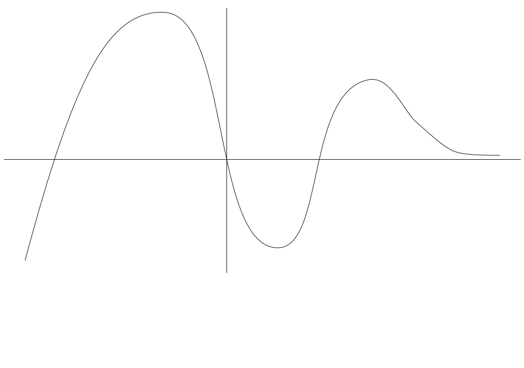
   (d)
   $$\lim_{x \to 0} \frac{\tan x}{x}$$

   (e)
   $$\lim_{x \to \pi} \frac{\tan x}{x}$$

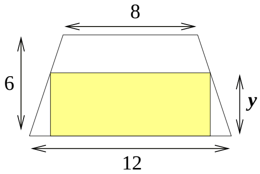
4. The figure shows the graph of the derivative f' of a function f.

   (a) Sketch the graph of f''.

   (b) Sketch the graph of f, given that $f(0) = 1$.



5. Find the dimensions of the rectangle of largest area that can be inscribed in an isosceles trapezoid of height 6 cm, with bases 8 cm and 12 cm.

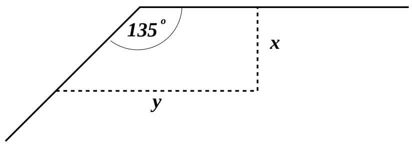


6. A water trough is 10 m long, has a cross-section in the shape of an isosceles trapezoid that is 30 cm wide at the bottom and 80 cm wide at the top. The height is 50 cm. If the trough is being filled with water at the rate of 0.2 $\mathrm{m}^3/\mathrm{min}$, how fast is the water level rising when the water is 30 cm deep?

7. Find $\dfrac{\mathrm{d}}{\mathrm{d}x} f(x)$, given that $\dfrac{\mathrm{d}}{\mathrm{d}x}[f(\mathrm{e}^x)] = x$.

8. Gravel is being dumped from a conveyor belt onto the ground at a rate of 30 $\mathrm{ft}^3/\mathrm{min}$. The gravel forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high?

9. **The two-sided pen.** A pen is to be constructed at the bend in a river using one mile of fence, as shown in the figure. What should x and y be to maximize the area?



10. Let
$$f(x) = \frac{x-1}{(2-x)^2}.$$
    (a) Find all horizontal and vertical asymptotes.
    (b) Find the intervals on which f is increasing, and the intervals on which f is decreasing. Find all critical points.
    (c) Find all local extrema.
    (d) Find the intervals on which the graph of f is concave up, and the intervals on which the graph of f is concave down. Find all points of inflection.
    (e) Sketch the graph of f, labelling all the features in parts (a)–(d).

11. (a) Show that between any two roots of a differentiable function f, there is a root of f'. Draw a picture.
    (b) An optional brain teaser: Show that the polynomial $f(x) = x^{101} + x^{51} + x - 1$ has exactly one root.

12. True or false? The graph of a quadratic function $f(x) = ax^2 + bx + c$ never has an inflection point. Explain in detail.
True or false? The graph of a cubic function $f(x) = ax^3 + bx^2 + cx + d$ always has exactly one inflection point. Explain in detail.
True or false? If a function f is continuous at a point a, then it is differentiable at a. Explain in detail.

True or false? If a function f is differentiable at a point a, then it is continuous at a. Explain in detail.

13. For what values of the constants a and b is (1,6) a point of inflection of the curve $y = x^3 + ax^2 + bx + 1$?

14. What special property does a continuous function on a closed interval domain have (which it does not necessarily have if the domain isn't closed)?

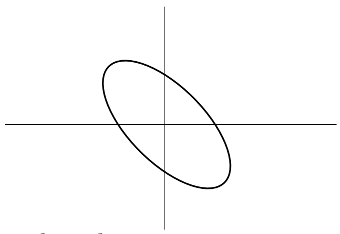
15. The long-distance running track shown in the figure consists of two parallel straight segments and two semicircular ends. The distance around the track is to be 2000 meters. Find the dimensions of the track so that the rectangular plot enclosed by the track is as large as possible.

16. Consider the function

$$f(x) = \begin{cases} x+2 & x \le -1 \\ x^2 & -1 < x < 1 \\ 2x & x \ge 1 \end{cases} .$$

Discuss the continuity and differentiability of this function at all points in its domain. Use appropriate limit computations or theorems to justify your answers.

17. Find the highest and lowest points on the curve $x^2 + xy + y^2 = 12$. The graph of this equation is the tilted ellipse in the figure.



Repeat this exercise, using the fact that the curve is parameterized by the equations

$$x(t) = 2\sin t + 2\sqrt{3}\cos t \qquad y(t) = 2\sin t - 2\sqrt{3}\cos t.$$

18. A family of ellipses is determined by the equation $4x^2 + y^2 = r^2$. There is one value of the constant r so that the line $y = 2 - 2x$ is a tangent line to the corresponding ellipse. Find the value of the constant r.