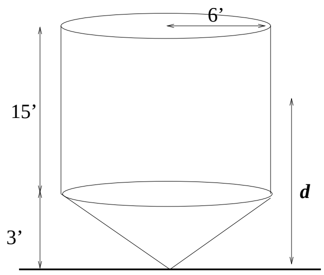
1. Use the definition of slope of a tangent line and the limit laws to find the slope of the tangent line to the graph of the function $f(x) = 8x^2 - x + 2$ at the point $(a, f(a))$. Write with complete detail.

2. An airplane is traveling in a path shaped like the graph of the parabolic function
$$f(x) = x^2 - x.$$
The slope of the tangent line to the path at any point $x = a$ is given by
$$f'(a) = 2a - 1.$$
The airplane starts at a point 5 miles west and 30 miles north of the origin, traveling in the increasing $x$ direction. At what point will the airplane be flying directly toward the point $(-1, 0)$?

3. Suppose $f$ is a continuous, differentiable function with the property that
$$|f(x)| \leq x^2 \quad \text{for all } x.$$
Use the Squeeze Law for limits to show that $f(0) = 0$ and that $f'(0) = 0$.

4. If $f$ is a continuous, differentiable function at the point $a$ and $g(x) = xf(x)$, use the definition of derivative to show that
$$g'(a) = af'(a) + f(a).$$

5. A tank used for portland cement consists of a cylinder mounted on top of a cone, with its vertex pointing downward. The cylinder has a height of 15 feet, both the cylinder and the cone have a radius of 6 feet, and the cone has a height of 3 feet.




   - Determine the volume of cement contained in the tank as a function of the depth $d$ of the cement.
   - What is the domain of this function?

6. Consider the pairs of parametric equations:

$$x = 2 - 3t \quad \text{and} \quad y = 7 - 6t$$

and

$$x = t - 1 \quad \text{and} \quad y = 1 + 2t.$$

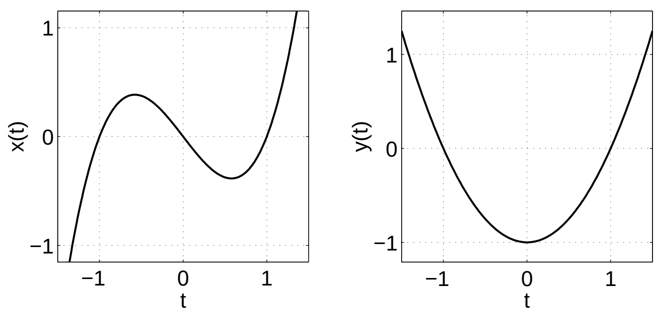
- Show that both of these pairs of equations produce the same line.
- What are the slope and $y$-intercept of this line?
- If, in the first pair of equations, $t$ varies in the interval $[0, 1]$, what interval must $t$ be in for the second pair of equations to cover the same line segment?

7. Is the function $f$ given by

$$f(x) = -2x + \sqrt[5]{3 - 2^{x+1}}$$

continuous at all points in its domain? Why or why not?
It is possible to deduce that there must be some number $c$ between 0 and 1 where $f(c) = 0$. Explain why we can make this conclusion.

8. Suppose the graphs of $x$ and $y$ as functions of $t$ look like




Describe how $x$ and $y$ increase and decrease as $t$ increases, and use these observations to draw a rough sketch of the parametric curve in the $xy$ plane, including an arrow indicating how the curve is traced out as the parameter $t$ increases.

9. Decide whether the given function has an inverse. Thoroughly defend your decision.

   (a) $P(x)$ is the cost, in cents, of mailing a letter that weighs $x$ ounces.

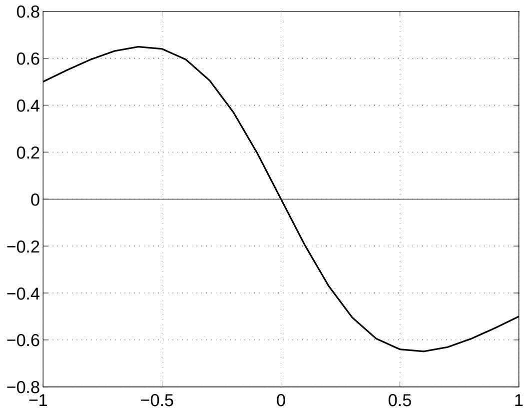
   (b) $C(t)$ is the total number of cars which have driven past a particular point along a highway during a specific day where $t$ represents the time, in hours, since midnight.

   (c) $V(x)$ is the volume of a cube whose side has length $x$.

   (d)
$$f(x) = (x-3)^2 + 1$$

   (e)
$$g(x) = \begin{cases} x^2+2 & x < 0 \\ 1-x & x \geq 0 \end{cases}$$

10. The graph of $y = f(x)$ is given by




   (a) On which intervals is $f$ increasing?

   (b) On which intervals is $f$ decreasing?

   (c) Sketch a graph which could represent $y = f'(x)$.

11. Explain why
$$\frac{(3x-2)(x-4)}{(x-4)} \neq (3x-2),$$
but
$$\lim_{x \to 4} \frac{(3x-2)(x-4)}{(x-4)} = 10.$$

12. Consider the function
$$f(x) = \ln(\ln x).$$

   (a) State the domain and range of $f$

   (b) Find $f^{-1}(x)$ and state its domain and range.