1. Find $f'(x)$ if it is known that
$$\frac{d}{dx}\left[f(2x)\right] = x^2.$$

2. (a) Given $g(x) = \sin x$, find $f^{(42)}(x)$.
   (b) Given $f(x) = x^2 \cos x$, find $f^{(42)}(x)$.

3. For what values of c does the equation $\ln x = cx^2$ have exactly one solution? (don't forget that c can be both positive and negative.)

4. A particle moves on a vertical line so that its coordinate at time t is $y = t^3 - 12t + 3$, $t \geq 0$.
   (a) Find the velocity and acceleration functions.
   (b) When is the particle moving upward and when is it moving downward?
   (c) Find the distance that the particle travels in the time interval $0 \leq t \leq 3$.
   (d) Sketch graphs of the position, velocity and acceleration functions for $0 \leq t \leq 3$.
   (e) Where is the graph of the position concave up? Concave down?
   When is the particle speeding up? When is it slowing down?

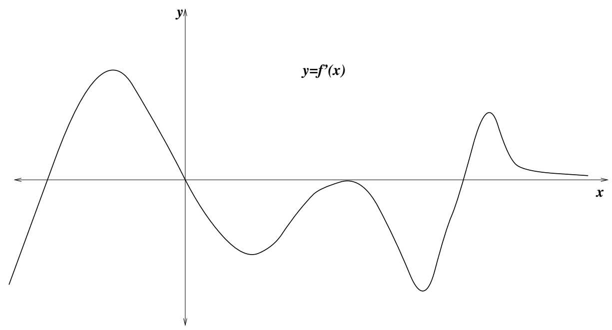
5. Use the definition of derivative to prove the **Reciprocal Rule**: If g is differentiable, then
$$\frac{d}{dx}\left(\frac{1}{g(x)}\right) = -\frac{g'(x)}{[g(x)]^2}.$$

6. Evaluate the following limits:
   (a)
$$\lim_{\theta \to 0} \frac{1 - \cos\theta}{\theta^2}$$
   (b)
$$\lim_{x \to \pi} \frac{e^{2\sin x} - 1}{x - \pi}$$
   (c)
$$\lim_{\theta \to 0} \frac{\theta + \sin\theta}{\tan\theta}$$

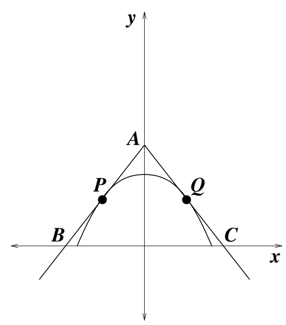
7. Let f be a function such that $f(2) = 1$ and whose derivative is known to be $f'(x) = \sqrt{x^5 + 4}$
   (a) Use a linear approximation to estimate the value of $f(2.03)$.
   (b) Will the exact value of $f(2.03)$ be less than or greater than your estimate? Clearly explain why.

8. An airplane is traveling in an elliptical holding pattern described by the parametric equations
$$x = 4\cos t \qquad y = 3\sin t,$$
where x and y have units of miles. The control tower is 5 miles east of the origin. At what point will the airplane by flying directly toward the control tower?

9. The figure shows the graph of the derivative f' of a function f.
   (a) Sketch the graph of f''.
   (b) Sketch the graph of f, given that $f(0) = 1$.



10. Find points P and Q on the parabola $y = 1 - x^2$ so that the triangle ABC formed by the x-axis and the tangent lines at P and Q is an equilateral triangle.



11. Water is flowing at a constant rate into a spherical tank. Let $V(t)$ be the volume of water in the tank and $H(t)$ be the height of the water in the tank at time t.
   (a) What are the meanings of $V'(t)$ and $H'(t)$? Are these derivatives positive, negative or zero?
   (b) Is $V''(t)$ positive, negative or zero? Explain.
   (c) Let t_1, t_2 and t_3 be times when the tank is one-quarter full, half full, and three-quarters full, respectively. Are the values $H''(t_1)$, $H''(t_2)$, $H''(t_3)$ positive, negative or zero? Why?

12. (a) Explain why $|x| = \sqrt{x^2}$ for all real numbers x.
   (b) Use part (a) and and the Chain Rule to show that
$$\frac{d}{dx}|x| = \frac{x}{|x|} \text{ for } x \neq 0.$$
   (c) If $f(x) = |\sin x|$, find $f'(x)$ and sketch the graph of f and f'. Where is f not differentiable?
   (d) If $g(x) = \sin|x|$, find $g'(x)$ and sketch the graph of g and g'. Where is g not differentiable?