Exam II Solutions

1.

a) $\int_0^{\pi} 2\pi x(\sin x + 2\sin x)\,dx$ $2\pi(\text{radius})(\text{height at value})$

b) $\int_0^{\pi} \pi(2+2\sin x)^2 - \pi(2-\sin x)^2\,dx$ $\pi(\text{outer curve})^2 - \pi(\text{inner curve})^2$

c) $\int_0^{\pi} \pi(2\sin x)^2\,dx$ * $\pi(r)^2$

*$2\sin x$ encompasses $\sin x$ while the shape spins $360°$ so $\int_0^{\pi}\pi(2\sin x)^2dx$ covers the entire area reached.

2 $L_1 = \int_0^{\pi} \sqrt{1+(\cos x)^2}\,dx$

$L_2 = \int_0^{\pi} \sqrt{1+(-2\cos x)^2}\,dx$

perimeter $= \int_0^{\pi} \sqrt{1+(\cos x)^2}\,dx + \int_0^{\pi}\sqrt{1+(-2\cos x)^2}\,dx$

3. According to the MVT, a function $F(x)$'s average rate of change, $F'(c)$ $[0 \le c \le 2]$

$= F'(c) = \frac{F(2)-F(0)}{2-0}$

$F(2) - F(0) = \int_0^2 F'(x)\,dx$

$F'(x) = f(x)$

$= f(c) = \frac{1}{2-0}\int_0^2 f(x)\,dx$

3. B.

$f(c) = \frac{1}{2} \cdot \int_0^2 x^2 + x\,dx$

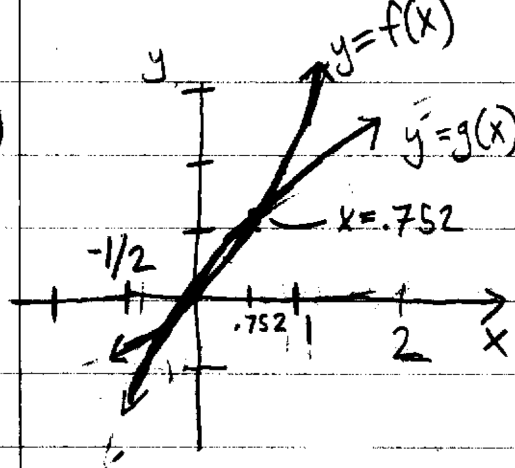
$= \frac{1}{2}\left[\frac{x^3}{3} + \frac{x^2}{2}\right]_0^2 = \frac{1}{2}\left(\left(\frac{8}{3}+2\right)-(0)\right)$

$= \frac{7}{3}$

$\frac{7}{3} = c^2 + c$

$0 = c^2 + c - \frac{7}{3}$

$\boxed{c = 1.1073}$

4a) 

$[-1/2, 2]$

$f(x) = e^x - 1$

$g(x) = 2\ln(x+1)$

4b) $[-1/2, 2]$

$$A = \int_{-1/2}^{0} [f(x) - g(x)] \cdot dx + \int_{0}^{.752} [g(x) - f(x)] \cdot dx + \int_{.752}^{2} [f(x) - g(x)] \cdot dx$$

so,

$$A = \int_{-1/2}^{0} [e^x - 1 - 2\ln(x+1)] \cdot dx + \int_{0}^{.752} [2\ln(x+1) - e^x + 1] \cdot dx + \int_{.752}^{2} [e^x - 1 - 2\ln(x+1)] \cdot dx$$

5 (a) not probability density function

because $\int_{-\infty}^{\infty} \frac{2}{1+e^{x}}$ is divergent

$\int_{-\infty}^{0} \frac{2}{1+e^{x}}\,dx = \infty$

(b) Probability density function because

$\int_{-\infty}^{0} \frac{1}{2}e^{x}\,dx + \int_{0}^{\infty} \frac{1}{2}e^{-x}\,dx = \frac{1}{2}e^{x}\Big]_{-\infty}^{0} + \frac{-1}{2}e^{-x}\Big]_{0}^{\infty}$

$\left[\frac{1}{2}\cdot 1 - \frac{1}{2}\cdot 0\right] + \left[\frac{-1}{2}\cdot 0 + \frac{1}{2}\cdot 1\right] = 1$

6.) a.) $a_n = \frac{2n}{n+1} \rightarrow \frac{2}{1+\frac{1}{n}}$

$$\lim_{n\to\infty} \frac{2}{1+\frac{1}{n}} = \frac{2}{1+0}$$

convergent.

$$\boxed{\lim_{n\to\infty} a_n = 2}$$

b.) $a_n = (n^2+n)^{1/n}$

$$\ln(a_n) = \frac{1}{n}\ln(n^2+n)$$

$$\lim_{n\to\infty} \ln(a_n) = \lim_{n\to\infty} \frac{\ln(n^2+n)}{n}$$

L'Hospitals Rule

$$= \lim_{n\to\infty} \frac{\frac{2n}{n^2+n}}{1} \rightarrow \frac{2}{2n+1}$$

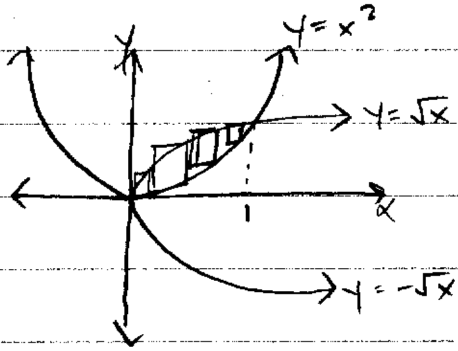
$$= \lim_{n\to\infty} \frac{2}{2n+1} = 0$$

$$\lim_{n\to\infty} \ln(a_n) = 0$$

$$\lim_{n\to\infty} e^{\ln(a_n)} = e^0$$

$$\boxed{\lim_{n\to\infty} a_n = 1}$$

convergent

7. 

$V = s^2 \qquad s = \sqrt{x} - x^2$

$V = \int_0^1 (\sqrt{x} - x^2)^2 \, dx$

$V = \int_0^1 x - 2x^2\sqrt{x} + x^4 \, dx$

$V = \int_0^1 x - 2\sqrt{x^5} + x^4 \, dx$

$V = \frac{x^2}{2} - 2\frac{x^{7/2}}{7/2} + \frac{x^5}{5} \Big]_0^1$

$V = \frac{1}{2}x^2 - 4/7\, x^{7/2} + 1/5\, x^5 \Big]_0^1$

$V = \left(1/2(1)^2 - 4/7(1)^{7/2} + 1/5(1)^5\right)$

$V = 1/2 - 4/7 + 1/5 = \boxed{9/70}$