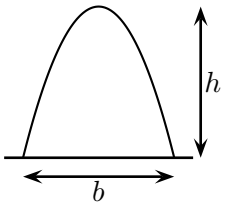
1. Use the limit definition of the definite integral to evaluate

$$\int_2^5 (4x - x^2)\, dx$$

2. There is a rule of thumb that the area of any parabolic arch is two-thirds the product of its height h and the length of its base b.



Confirm this formula by using the Fundamental Theorem of Calculus on a suitable function to calculate the area in question.

3. If f is a continuous function, find $f(4)$ if

$$x \sin \pi x = \int_0^{x^2} f(t)dt$$

4. Steve and Rick are having an argument as to the value of

$$\int \sec^2(x) \tan x \, dx.$$

Steve makes the substitution $u = \sec x$ and Rick uses the substitution $v = \tan x$. Both substitutions seem to work, but they arrive at completely different-looking answers. What's going on?

5. The average y-value of a function $y = f(x)$ over the interval $[a, b]$ is equal to the number

$$f_{\text{ave}} = \frac{1}{b-a} \int_a^b f(x)\, dx.$$

[We will see later where this formula comes from and why it gives the average y-value.]

   (a) Find the average values of $f(x) = x$, $f(x) = x^2$, $f(x) = x^3$, and $f(x) = x^4$ on the interval $[0, 1]$.

   (b) From the pattern that is established in the previous part, what would you guess is the average value of $f(x) = x^n$ on the interval $[0, 1]$ for any integer $n \geq 1$? Justify your guess.

   (c) What does the answer to the second part imply about the average value of $f(x) = x^n$ as n gets larger and larger? Can you explain this from the graphs of $f(x) = x^n$

6. For each of the following indefinite integrals, determine which technique — substitution or integration by parts — will succeed in finding an antiderivative. [NOTE: You do not have to find the integrals themselves, just do enough computation to be *sure* that the technique you chose will work.]

(a) $\displaystyle\int x^3 \ln x \, dx$

(b) $\displaystyle\int \frac{(\ln x)^3}{x} \, dx$

(c) $\displaystyle\int x(\ln x)^3 \, dx$

(d) $\displaystyle\int \frac{\ln x}{x^3} \, dx$

(e) $\displaystyle\int \frac{\ln x^3}{x} \, dx$

(f) $\displaystyle\int \frac{\ln 3x}{x} \, dx$

(g) $\displaystyle\int \frac{\ln x^3}{x^3} \, dx$

(h) $\displaystyle\int (\ln x)^3 \, dx$

7. Let f be a continuous function with antiderivative F on the interval $[a, b]$. Let c be any point in the interval. State whether the following are true or false. If false, then given an example to show why it is false and correct the statement if possible. If true, explain why.

(a) $\displaystyle\int_a^b f(x) \, dx = \int_a^c f(x) \, dx = \int_c^b f(x) \, dx$

(b) $\displaystyle\int_a^b F(x) \, dx = f(b) - f(a)$

(c) $\displaystyle\int_a^b f(x) \, dx \geq 0$

(d) $\displaystyle\int_a^b cf(x) \, dx = c[F(b) - F(a)]$

8. Show that the area enclosed by the graph of the parabola

$$y = f(x) = \frac{2}{a^2}x - \frac{1}{a^3}x^2, \qquad a > 0,$$

and the x-axis has an area independent of a. How large is the area?