Distribution of Means of Random Variables (Due Friday, January 16)

Goals

- 1. Practice simulating and describing the behavior of independent random variables.
- 2. Explore the properties of distributions obtained by averaging independent random variables.
- 3. See the Central Limit Theorem in action through simulation experiments.

Problems

For each of the following, submit the required R commands along with your answers.

- 1. Let X be a random variable with Uniform (0,1) distribution and let Y be a random variable with an Exponential (1) distribution. Use R to generate 500 observations on X and 500 observations on Y. Obtain histograms for each of the two sets of 500 observations and confirm that they have the characteristics (shape, mean, and variance) that you expect. Also, comment on how the shapes of these histograms compares with what you would expect if you obtained one million observations on each random variable rather than just 500.
- Use R to simulate the sampling distribution of the sample mean X for samples of size 2 from a Uniform (0,1) distribution. Repeat this simulating using the following sample sizes: 5, 10, and 30. Include your R script, a histogram of the sampling distribution, and summary statistics.
- 3. Repeat the previous problem but use the Exponential(1) distribution instead of the Uniform (0,1) distribution.
- 4. Repeat the previous problem but use a non-normal distribution of your choosing, e.g., Chi-Square with 5 degrees of freedom.
- 5. Summarize your findings from these distributions. Describe the shapes of the distributions, comment on how the shape changes as n increases and comment on similarities and differences across the three distributions.