Distribution of Means of Random Variables (Due Friday, January 23)

Goals

- 1. Practice simulating and describing the behavior of independent random variables.
- 2. Explore the properties of statistics and distributions related to the normal distribution.

Problems

For each of the following, submit the required R commands along with your answers. For the first, a script has been provided for you (see the web site at http://kzoo.edu/enordmoe/math365).

- Set the seed equal to 48 and simulate a χ²₃ distribution by summing the squares of three simulated standard normal random variables, each having length 500. Create a density histogram (prob=T) of the simulated χ²₃ random variable. Superimpose the theoretical χ²₃ density over the histogram. Comment on how well the simulation fits with what the theory of Section 6.4 would predict.
- 2. Given $X \sim N(0, 1)$, $Y \sim N(2, 2)$ and $Z \sim N(4, 3)$, what is the distribution of W = X + Y + Z? Set the seed equal to 365 and simulate 500 samples, each of size 1 for X, Y, and Z. Add the values in the three vectors obtain W's empirical distribution. Create a density histogram of the simulated values of W and superimpose the theoretical density of W.
- 3. Verify empirically that

$$\frac{N(0,1)}{\sqrt{\chi_5^2/5}} \sim t_5$$

by setting the seed equal to 36 and generating a sample of size 1000 from a N(0, 1) distribution. Generate another sample of size 1000 from a χ_5^2 distribution. Have R perform the appropriate calculation to arrive at the simulated sampling distribution. Create a density histogram of the results and superimpose a theoretical t_5 density.

4. Verify empirically that

$$\chi_3^2 + \chi_2^2 \sim \chi_5^2$$

by setting the seed equal to 79 and generating a sample of size 1000 from a χ_3^2 distribution. Generate another sample of size 1000 from a χ_2^2 distribution. Have R perform the appropriate calculation to arrive at the simulated sampling distribution. Create a density histogram of the results and superimpose a theoretical χ_5^2 density. **Extra credit problem:** Set the seed equal to 10 and simulate 1000 random samples of size $n_X = 65$ from a $N(4, \sqrt{2})$ distribution and 1000 random samples of size $n_y = 90$ from a $N(5, \sqrt{3})$ distribution. Verify that the simulated statistic $\frac{S_X^2/\sigma_X^2}{S_Y^2/\sigma_Y^2}$ actually follows an $F_{64,89}$ distribution by plotting a histogram with the appropriate density curve superimposed.