Distribution of Means of Random Variables (Due Friday, February 6)

Goals

- 1. Explore the properties of confidence intervals.
- 2. Explore sensitivity of coverage probabilities to (i) sample size and (ii) the shape of the original distribution

Problems

For each of the following, submit the required R commands along with your answers. A script has been provided for you (see the web site at http://kzoo.edu/enordmoe/math365).

1. Use the script provided for this lab to carry out a simulation experiment to investigate the coverage properties (percent of intervals that contain the true population mean μ) of the confidence interval

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \quad . \tag{1}$$

Set the number of samples to be 100, the mean to be 1 and the variance to be 1. Carry out simulations for three different sample sizes: n = 3, n = 10, and n = 30. Make a summary table of your reports including the number of the random intervals out of 100 that contained the true mean μ in each case.

- 2. Repeat the previous simulation experiment but this time modify the script so that the sample is drawn from an Exponential distribution with mean and variance both equal to 1.
- 3. Repeat the previous simulation experiment again but this time modify the script so that the sample is drawn from a Uniform [0, 1] population. Note: Be sure to notice that the mean and standard deviation are no longer 1. Use the sample mean and population standard deviation (from the Uniform distribution with A = 0 and B = 1) to compute the intervals as in (1). To calculate the proportion that contain the true population mean, be sure to use the mean derived from the Uniform distribution (not the sample mean).
- 4. Summarize your findings from these simulation experiments. Describe how the coverage properties of this confidence interval change as n increases and comment on similarities and differences across the distributions.