Lagrange Multipliers for Real

November 3, 2021

The second Calc 3 project. This project will be worth 1.5 as much as the first project. It is unclear at this point if there will be a third (smaller) project. Note that this project has two goals. The first is to provide an application of the concepts we have been studying beyond a one-step homework problem. The second is to provide an opportunity to communicate mathematics. (That is, this project is designed to help us move up the ladder in Bloom's Taxonomy.)

You may complete this project with a partner or you may work independently. If you work with a partner (just one!), only one write up is submitted for the pair. If you choose to work as a pair, both of you will receive the same grade. The paper is to be type-written, but the mathematics symbols may be written in by hand. As in the first project, how well your report is received will be dependent on how correct the mathematics is as well as how well written it is.

This is adapted from the hydro-turbine project on pages 821-822 of our text which was adapted from a project written by Phil Straffin of Beloit College with technical input from Harry Bard of the Great Northern Paper Co. The functions you need are on those pages in our text.

The Great Northern Paper Company became, in part, the Katahdin Paper Company in 2017. Using real measurements, the power output of some of its hydro-electric turbines were modelled, and they appear in our text on p821. You are being hired by the company as a consultant to help them utilize this data. In particular, you are expected to write a formal report for the CEO of the company. Your report should be structured so as to include an introduction to the main goals of your report (because the CEO reads lots of different reports each day), the completion of the tasks at hand, an analysis of that work, and a summary. While the completion of the tasks will be technical, you can expect that the CEO will pass this off to her chief engineer to make sure they make sense. Thus, the introduction, analysis and summary should be clear, convincing, and readable even without a careful reading of the technicalities. Your report, in particular, should use full sentences. The technicalities should include enough exposition so that the chief engineer, who is not an expert in this method of optimization but who does have a math background, can be convinced that you are correct.

You are tasked with allocating water flow through three turbines so as to maximize the total energy they produce and to begin to address the question of whether using one or two turbines might sometimes be more efficient. Specifically, you are not being paid to:

- maximize (using Lagrange multipliers) the total energy production $K = KW_1 + KW_2 + KW_3$ of the three main turbines when the total flow of water Q_T through the turbines is 2500 cfs (cubic feet per second).
- maximize (using Lagrange multipliers) the total energy production K of the three main turbines when the total flow of water Q_T through the turbines in terms of Q_T and the range of values of Q_T for which your work applies (the allowable flows of operation for each turbine are in our text).
- graph the efficiency KW_i/Q_i (ie, kilowatts produced per unit of water flow) of each of the three turbines (as a function of the water flow) over the allowable flow of operation.
- maximize (using Lagrange multipliers) the total energy production when just turbines 1 and 3 are used.
- analyze which turbine should run if only one turbine is to be used.
- provide analysis across various levels of water flow (250cfs to 3000cfs) as to which combination of turbines seems most advantageous to run. While your data is not quite enough to provide a full answer here, you do have quite a lot of information. Make clear in your report where your data support your conclusions and where you are using your consultant expertise to infer.

These tasks do not need to be presented in your report in the order in which they are listed here.

You might find it useful (but it is not necessary) to recall that while the maximum values will be different for a function and that function divided by a constant (say, $170 - 1.6 \cdot 10^{-6} Q_T^2$), *where* the maximum occurs will be the same. You might also find it advantageous to not round your answers too much, and to graph the three efficiency curves together in a single graph (where, indeed, one curve will extend a little longer than the others).

This project is due in Moodle by **8:15am Friday, Week 9**. Both members of a team should be listed, and both are responsible for making sure the deadline is met.