

T. Houdini
135 Prestidigitator St.
Handcuff, MI 59055

Math 113 Students
Kalamazoo College
Kalamazoo, MI 49006

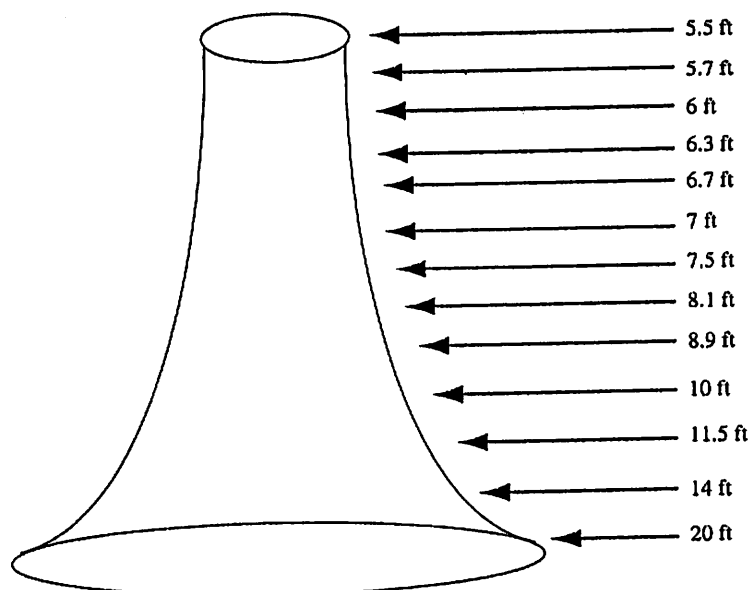
Dear Calculus Students:

I have decided to continue the family business established by my grandfather, and I need some help planning one of the escapes that I am including in my inaugural tour. When I went looking for help, your enterprising and resourceful professor referred me to you.

I will be locked in chains and have my feet shackled to the top of a stool which is attached to the bottom of a giant tank that looks vaguely like a laboratory flask. The flask will be filled with water (at a constant rate of 500 gallons per minute), and after much practice out of the water, I have determined that it will take me exactly 10 minutes to escape from the chains.

I have a flair for the dramatic, so I would like to escape from the shackles at the exact instant that the water reaches the top of my head. I need your help in determining how tall the stool should be. Also, I want to monitor the rise of the water during the escape, so at any time after the water begins flowing, I want to know how high the water is in the tank and how fast the water is rising. While I am fairly accomplished at holding my breath under water, I would like to know how long I will have to hold my breath during this last part of the stunt.

I've included a sketch of the tank below, which gives the diameter of the tank at 1 foot intervals.



After consulting with your enterprising and resourceful professor, he suggested that you might be interested to know that I am 5 feet 9 inches tall, and I'm pretty skinny so that you can ignore both my volume and the volume of the stool in your analysis.

I realize that this is a busy time of year for you, but I would greatly appreciate an answer by Monday of Week 8, since my tour opens at the State Theatre the following weekend.

Yours sincerely,
T. Houdini

A Few Comments From Your Enterprising and Resourceful Professor

After consulting with T. Houdini, I have a few suggestions which may help you get started:

- A gallon is equal to 0.13368 cubic feet.
- The tank looks very much like a solid of rotation, and the function $f(z) = \frac{10}{\sqrt{z+1}}$ looks like it might be a good model for the tank.
- You should be able to express the volume of the water in the tank as a function of the height of the water above ground level. What is the volume when the water reaches the top of Houdini's head? Once you have done this, you should be able to determine the height of the stool.
- You can think of the volume and the height of the water as functions of time. You should be able to easily find an expression for the $V(t)$ and then use your expression for the volume in terms of the height to solve for $h(t)$.