1. Evaluate each limit. NOTE: You can't use L'Hospital's rule because we haven't learned that!

   (a) $\lim_{\theta \to 0} \dfrac{\theta^2 + \tan(\theta)}{\sin(\theta)}$

   (b) $\lim_{x \to 0} \dfrac{\sin^2(2x)}{3x^2}$

   (c) $\lim_{x \to 1} \dfrac{\arctan(x) - \frac{\pi}{4}}{x - 1}$

2. A round bottomed beaker (pictured below) is initially empty and is filled with water at a constant rate.

   (a) Let $h(t)$ be the height of water in the hourglass as a function of time. Give a sketch of the graph of $h(t)$.

   Make sure your sketch is correct in terms of concavity.

   (b) Sketch $h'(t)$.

   (c) Let t_1 represent the time when the hourglass is half full. Is $h''(t_1)$ positive, negative or zero?

3. Consider the curve given by parametric equations

$$x = \frac{2t}{t+1} \qquad\qquad y = e^{\sqrt{t}}$$

   (a) Find $\frac{dy}{dx}$ in terms of t. You don't need to simplify your answer.

   (b) Find the equation of the tangent line to the curve at the point $(1, e)$.

Let g be a differentiable function with

$$g(4) = 1 \qquad\qquad g'(4) = -1 \qquad\qquad g''(4) = -5$$

and let

$$H(x) = g(x^2).$$

Lebron James and Steph Curry are having a heated discussion about the function H.

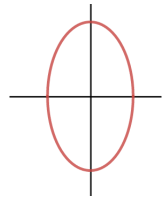
   (a) Lebron claims the function H is increasing at the point where $x = 2$. Determine if he is correct. Show your work.

   (b) Steph claims that H is concave down at the point where $x = 2$. Is he right? Show your work.

4. (a) Find the first **five** derivatives of $f(x) = x\sin(x)$. At each stage simplify your answer.

   (b) Use the pattern you observe in your work from part (a) to determine $f^{(30)}(x)$.

5. Find all the points on the ellipse $3x^2 + y^2 = 12$ where the slope of the tangent line is -1.



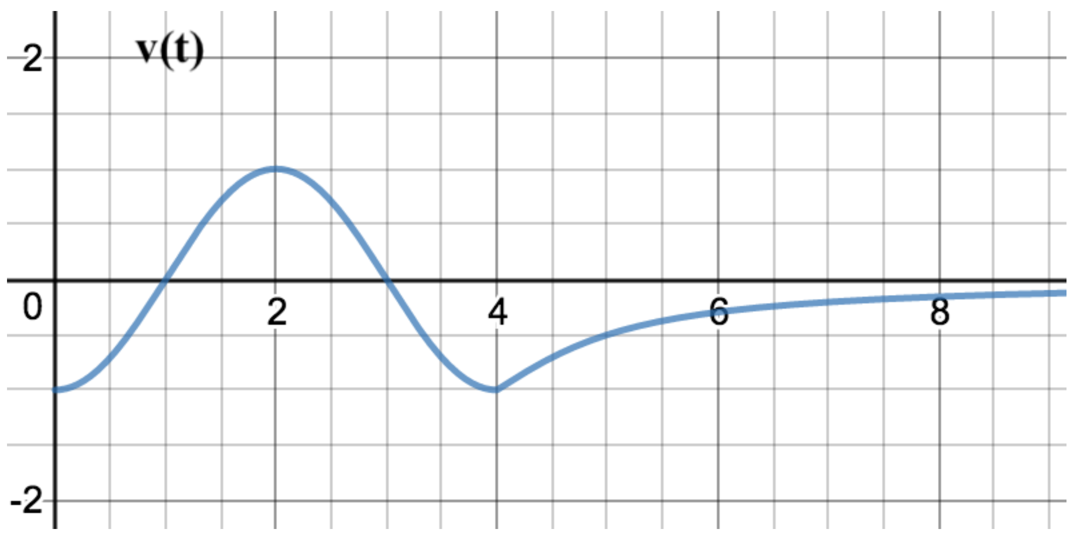
6. Consider the curve given by parametric equations

$$x = \frac{4t}{t+1} \qquad\qquad y = \ln(\sqrt{t})$$

   (a) Find $\frac{dy}{dx}$ in terms of t. You don't need to simplify your answer.

   (b) Find the equation of the tangent line to the curve at the point $(2, 0)$.

7. The following is a graph of the **velocity** of a particle that is moving along a straight line. (Time t is measured in seconds, velocity is in meters per second, and $0 \le t \le 9$.)



   (a) Give a sketch of the acceleration function, $a(t)$.

   (b) We also know that at time zero the particle is at position zero.

   Give a **sketch** of the position function $s(t)$. Make sure your sketch is **correct** in terms of **concavity**.

   (c) When is the particle moving in the positive direction?