November 5, 2008

1. Find f' if it is known that

$$\frac{d}{dx}\left[f(2x)\right] = x^2.$$

- 2. (a) Given  $g(x) = \sin x$ , find  $g^{(42)}(x)$ .
  - (b) Given h(x) = xf(x), find  $h^{(42)}(x)$ .
  - (c) If  $l(x) = x \sin x$ , use parts (a) and (b) to find  $l^{(42)}(x)$ .
- 3. For what values of the constant c does the equation  $\ln x = cx^2$  have exactly one solution? (Don't forget that c can be both positive and negative. Draw graphs for several values of c.)
- 4. A particle moves on a vertical line so that its coordinate at time t is  $y = t^3 12t + 3$ ,  $t \ge 0$ .
  - (a) Find the velocity and acceleration functions.
  - (b) When is the particle moving upward and when is it moving downward?
  - (c) Find the total distance that the particle travels in the time interval  $0 \le t \le 3$ .
  - (d) Sketch graphs of the position, velocity and acceleration functions for  $0 \le t \le 3$ .
  - (e) Where is the graph of the position concave up? Concave down? When is the particle speeding up? When is it slowing down?
- 5. Use the definition of derivative to prove the **Reciprocal Rule**: If g is differentiable, then

$$\frac{d}{dx}\left(\frac{1}{g(x)}\right) = -\frac{g'(x)}{[g(x)]^2}.$$

6. Evaluate the following limits:

(a)

(b)

$$\lim_{x \to \pi} \frac{e^{2\sin x} - 1}{x - \pi}$$

$$\lim_{\theta \to 0} \frac{1 - \cos \theta}{\theta^2}$$

(c)

$$\lim_{\theta \to 0} \frac{\theta + \sin \theta}{\tan \theta}$$

7. An airplane is traveling in an elliptical holding pattern described by the parametric equations

 $x = 4\cos t \qquad y = 3\sin t,$ 

where x and y have units of miles. The control tower is 5 miles east of the origin. At what point will the airplane by flying directly toward the control tower?

- 8. The figure shows the graph of the derivative f' of a function f.
  - (a) Sketch the graph of f''.
  - (b) Sketch the graph of f, given that f(0) = 1.



9. Find points P and Q on the parabola  $y = 1 - x^2$  so that the triangle ABC formed by the x-axis and the tangent lines at P and Q is an equilateral triangle.



- 10. Water is flowing at a constant rate into a spherical tank. Let V(t) be the volume of water in the tank and H(t) be the height of the water in the tank at time t.
  - (a) What are the meanings of V'(t) and H'(t)? Are these derivatives positive, negative or zero?
  - (b) Is V''(t) positive, negative or zero? Explain.
  - (c) Let  $t_1, t_2$  and  $t_3$  be times when the tank is one-quarter full, half full, and three-quarters full, respectively. Are the values  $H''(t_1), H''(t_2), H''(t_3)$  positive, negative or zero? Why?
- 11. (a) Explain why  $|x| = \sqrt{x^2}$  for all real numbers x.
  - (b) Use part (a) and and the Chain Rule to show that

$$\frac{d}{dx}|x| = \frac{x}{|x|}$$

- (c) If  $f(x) = |\sin x|$ , find f'(x) and sketch the graph of f and f'. Where is f not differentiable?
- (d) If  $g(x) = \sin |x|$ , find g'(x) and sketch the graph of g and g'. Where is g not differentiable?