

Deriving Kepler's Laws

September 28, 2021

Your first Calc 3 project!

For this first project, you may complete this project with a partner or you may work independently. If you work as a team, only one write up is submitted for the pair. If you choose to work as a team, both of you are responsible for making sure the deadline is met, and both of you will receive the same grade.

The project? Mathematically, your assignment is to work through Problems 1 -3 of the Kepler's Laws project on p726-727 of our textbook. It is also to communicate this work in a short paper that could be read by students. That is, you should write up your work as if Dr. Stewart himself asked you to write a few textbook pages that derive these laws so he could replace this project with your amazing write up. Thus, the readers of your work have not been asked these direct questions, but they have read the textbook up to this point.

In particular, your write up should NOT be: Problem 1) some math computation Problem 2) some more math computation etc. Instead, your work should consist of full sentences and full paragraphs, with the math incorporated into those sentences. It should be well-written - something you would be proud to show your first year seminar writing professor. **It should be typewritten, although the math can be written in by hand.** While you can assume your readers know as much calculus as you do before you start on this project, (ie, that they have read the textbook up to this point), you probably want to remind your reader of the notation that is specific from the derivation of the first law as you use it. In particular, your write up should explain those things that you had trouble figuring out so that your readers don't have the same trouble.

To be clear, how well this project is received (ie, the grade it earns) will depend not only on correctly doing the mathematics, but also on the quality of the exposition. An A paper is one that any Calculus 3 student at this point in the course can read and understand as easily as the rest of the textbook. A D paper is one which contains the correct mathematics computations, but submits those computations without much explanation. In between that, well, maybe you've introduced notation without defining it. Maybe you have typos and non-sentences, and other grammatical mistakes, maybe you have some mathematical errors, maybe you write sentences and incorporate your work but don't actually ever state what problem your paper is trying to address or how the steps you do are accomplishing this. These are things to try to avoid.

One thing it will be helpful to know is that the area of a sector of an ellipse (where the origin is at one of the foci, as it is here) is

$$\int_{\theta_1}^{\theta_2} \frac{1}{2} |\mathbf{r}(\theta)|^2 d\theta$$

where $\mathbf{r}(\theta)$ is the equation of the ellipse. Of course, if θ is a function of t , then you can do this with respect to t (just don't forget the chain rule!) You will also find it helpful to know two facts about ellipses: that when the major axis has length $2a$ and the minor axis has length $2b$, the eccentricity e can be written as

$$e = \frac{\sqrt{a^2 - b^2}}{a} \text{ and that } a = \frac{ed}{1 - e^2}$$

where d is the directrix.

This project is due in Moodle by **5pm Friday, Week 4**. Both members of a team should be listed, and both are responsible for making sure the deadline is met.