Abstract Algebra II

Problem Set 4
May 7, 2021

This problem set is due in Moodle at noon on Friday, Week 8. If that is announced as DOGL, the new due date is Sunday, May 23 at 9am.

Here are some useful definitions:

An $R$-module $M$ is *irreducible* if $M \neq 0$, and 0 and $M$ are the only submodules of $M$.

A *division ring* is a ring with unity, such that every nonzero element has a multiplicative inverse (such a ring is almost a field - it might not be commutative).

**Problem 1** Let $M$ be an $R$-module. Prove that $M$ is irreducible if and only if $M \neq 0$ and $M$ is a cyclic module where every nonzero element is a generator.

**Problem 2.** **(a)** Show that if $M_1$ and $M_2$ are irreducible $R$-modules, then any nonzero $R$-module homomorphism $\phi : M_1 \to M_2$ is an isomorphism.

**(b)** Show that if $M$ is irreducible, then $End_R(M)$ is a division ring.

**Problem 3.** Let $M$ be an $R$-module. Show that $M$ is irreducible if and only if $M \cong R/I$ where $I$ is a maximal ideal of $R$.