We have been working with vectors in $\mathbb{R}^n$.
$\mathbb{R}^n$ is our most important example of a <u>vector space</u>: • we can add vectors in $\mathbb{R}^n$ <u>and stay in $\mathbb{R}^n$</u>

• we can multiply vectors in $\mathbb{R}^n$ by scalars <u>and stay in $\mathbb{R}^n$</u>.

A <u>subspace</u> of $\mathbb{R}^n$ is a subset of vectors that has those same properties.

<u>Ex</u>: • Lines in $\mathbb{R}^n$ are examples of subspaces — But only if they pass through the origin.

• planes that pass through the origin in $\mathbb{R}^n$ are examples too.

• if $v_1, \dots v_k$ are vectors in $\mathbb{R}^n$, then $\mathrm{Span}\{v_1, \dots v_k\}$ is a subspace.

Two special examples: Fix a Matrix A, $m \times n$ $\quad m\underset{n}{\boxed{A}}$

• $N(A) = \{ x \text{ in } \mathbb{R}^n \mid Ax = \vec{0} \}$

$C(A) = \{ x \in \mathbb{R}^m \mid x \text{ is a linear combination of cols of } A \}$

Ex $A = \begin{bmatrix} 1 & 2 & 3 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 2 & 3 & 0 & 0 \end{bmatrix}$

$$N(A) = \left\{ x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 3/2 \\ 0 \\ -\frac{1}{2} \\ 1 \\ 0 \end{bmatrix} \;\middle|\; x_2, x_4 \text{ are real \#} \right\}.$$

$$C(A) = \text{Span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$

<u>Def</u>: A <u>basis</u> for a vector space is a

** collection of vectors which span the vector space and are linearly independent.

<u>Def</u>: The <u>dimension</u> of a vector space is the number of vectors in a basis for it.

(The dimension of $C(A)$ is often called the rank of A).

basis for $N(A)$ above: $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3/2 \\ 0 \\ -\frac{1}{2} \\ 1 \\ 0 \end{bmatrix} \right\}$

basis for $C(A)$ above: $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}.$