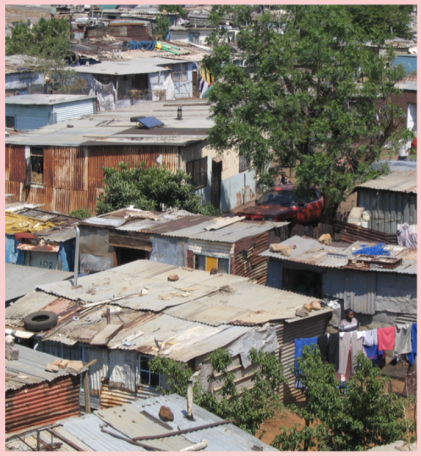
Modeling Wealth Inequality

How the rich get richer and the poor get poorer

Jan Tobochnik, Wolfgang Christian, and Harvey Gould



Preface

This pdf is a static version of an electronic publication that includes simulations which can be run on mobile devices or computers. The electronic version is available from the AAPT-ComPADRE website <http://www.compadre.org/osp/items/detail.cfm?ID=13337> in various formats including web pages with embedded simulations, a ComPADRE book <http://www.compadre.org/books/ModelingWealth>, and an ePub file that can be loaded into ebook readers that support ePub 3. (Unfortunately, some popular ebook readers, such as the Kindle and the Kindle App, do not yet support ePub 3.) The Readium plugin for the Google Chrome browser, the Adobe Digital Editions reader, the Kobo app, and the iBooks reader provide good ePub 3 support.

The ePub 3 version can also be freely downloaded from the iTunes store <https://itunes.apple.com/us/book/modeling-wealth-inequality/id1064963964?ls=1&mt=11> into the Apple iBooks reader on OS X computers and iOS mobile devices.

The use of JavaScript allows EPub 3 readers and web browsers to reproduce the results discussed in the following as well as to obtain results with alternative parameters. Native iOS and Android apps are in preparation to allow users to run these simulations on mobile devices without the use of an ePub reader or an Internet connection.

1 Why is there wealth inequality?

Most present-day societies exhibit large wealth inequalities. In particular, wealth inequality in the United States has increased significantly since the early 1970s, and the gap between the richest 1% and the remaining 99% is the greatest since the 1920s. The existence of wealth inequality has become so obvious that its existence is no longer in dispute.

What are the reasons for the widening gap between the very rich and everybody else? Why does wealth inequality occur? There are likely many political, economic, and cultural reasons. We could learn how some people became very wealthy and what government policies contribute to greater or lesser inequality.

We will learn in the following that simple models that do not include any specific details of economic activity show that inequality is a general and natural occurrence and is very difficult to prevent. Unlike climate models, for example, which require much background in science and very powerful computers, the models we will discuss can be simulated on a smart phone, tablet, or a laptop. We encourage you to play with the simulations and explore their results.

2 The Model

The goal of the model we will discuss is not to reproduce the specifics of particular societies, but to provide insight into the general behavior of economic activity. The beauty of the model is how little is needed to produce results that help us understand why wealth inequality occurs. The model is based on many exchanges of wealth between people, companies, or countries, which we will refer to as agents. The key assumption is that the wealth exchanged between any two agents is a percentage of the wealth of the poorer agent, which is similar to real-life economic transactions.

The model can be described by the following steps (or algorithm).

1. Specify the number of agents and give each a certain amount of initial wealth.
2. Choose two agents at random regardless of their wealth.
3. Determine which of the two agents has the smaller wealth, and take a percentage of the smaller wealth as the amount to be gained or lost in the transaction. For example, if the poorer agent has \$100 and the percentage is 10%, this amount is \$10.
4. Choose at random which agent gains and which loses. In our example, if the richer agent has \$1000, then after the transaction there are two possibilities that can be chosen by the flip of a coin: either the richer agent now has \$1010 and the poorer has \$90, or the richer has \$990 and the poorer has \$110.

5. Repeat steps 2–4 many times and look at how the wealth becomes redistributed among the agents.

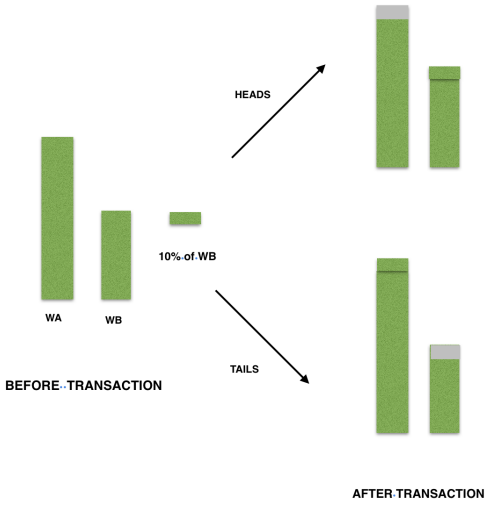


Figure 1: Illustration of the wealth exchange algorithm. This simulation is available at <http://www.compadre.org/osp/EJSS/3901/model2/41.htm?F=1>

.

Agents represent any entity that can hold or exchange wealth. We will assume that "wealth" includes currency or anything that can be bought or sold with currency such as real estate, cars, art, stock, commodities, or even factories.

3 Wealth distributions from the model

The key idea of the model is that usually one agent gains and one agent loses when they engage in economic activity. If you buy something, you give some of your wealth to the store in the form of money and you receive a product in exchange, which adds to your wealth. Usually, the store gains a profit from this exchange because the product is worth less than the money you paid for it. Thus, the store owner, after paying wages and other business costs, will usually have increased his or her personal wealth and you will have decreased yours. Occasionally the store might have a sale or promotion and sell a product for less than its cost. In this case you gain and the store loses wealth. A similar gain and loss occurs when paying wages and salaries and from buying and selling stocks. There is always somebody who gains and somebody who loses.

In brief, wealth is exchanged in many economic activities, and this economic activity is captured in part by the model. Exchanges of equal value do not change the wealth of either agent and thus do not need to be included in the model. We will see that the details of the model, such as the number of agents, the initial wealth distribution, and the percentage of the minimum wealth that is exchanged do not effect the ultimate results. These details just change how long it takes to obtain the final distribution of wealth. For simplicity, we will choose the default value of the percentage exchanged to be 10% of the wealth of the poorer agent.

Suppose that we consider a system of one hundred agents and give each agent the same amount of wealth. Before exploring the results of implementing the model, what do you expect to happen? After many exchanges, will the agents have roughly the same amount of wealth as they did initially? Will there be a well known distribution such as a bell shaped curve? Will there be a distribution of wealth among the agents such that we might be able to distinguish between the wealthy, the poor, and maybe a middle class?

4 What does the model predict?

Look at the next few figures to see what happens to the wealth exchange model that we have just described. A wealth of 1.0 on a plot corresponds to $100,000. The agents are labeled from 1 to 100 on the horizontal axis.

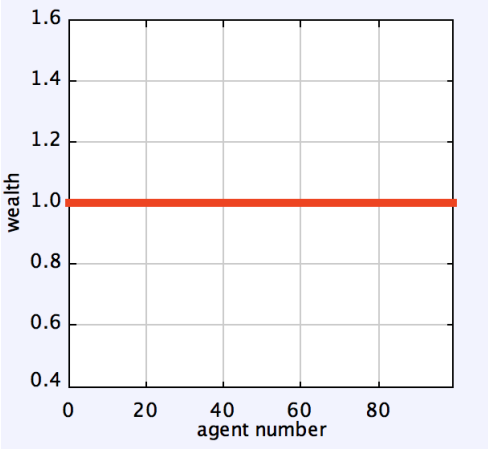


Figure 2: Initially all agents have the same wealth. This simulation is available at <http://www.compadre.org/osp/EJSS/3901/model3/42.htm?F=1>.

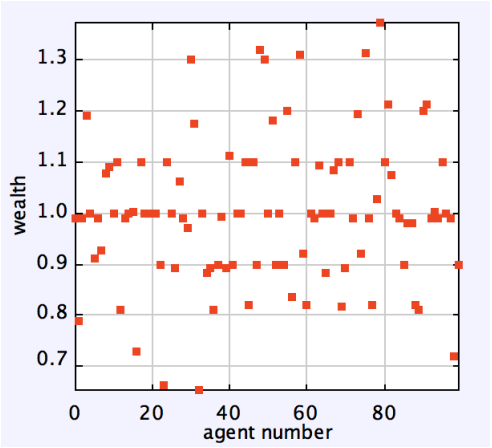


Figure 3: The wealth distribution after 100 transactions. This simulation is available at <http://www.compadre.org/osp/EJSS/3901/model3/42.htm?F=1>.

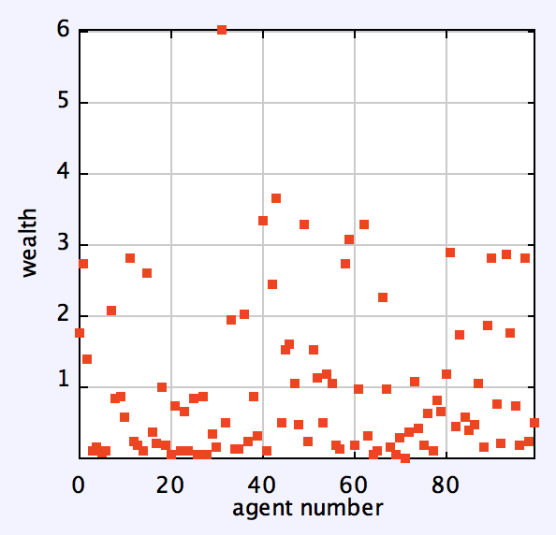


Figure 4: The wealth distribution after 10,000 transactions.This simulation is available at <http://www.compadre.org/osp/EJSS/3901/model3/42.htm?F=1>.

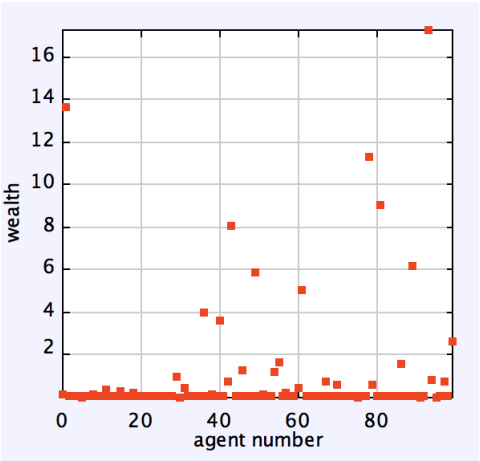


Figure 5: The wealth distribution after 100,000 transactions.This simulation is available at <http://www.compadre.org/osp/EJSS/3901/model3/42.htm?F=1>.

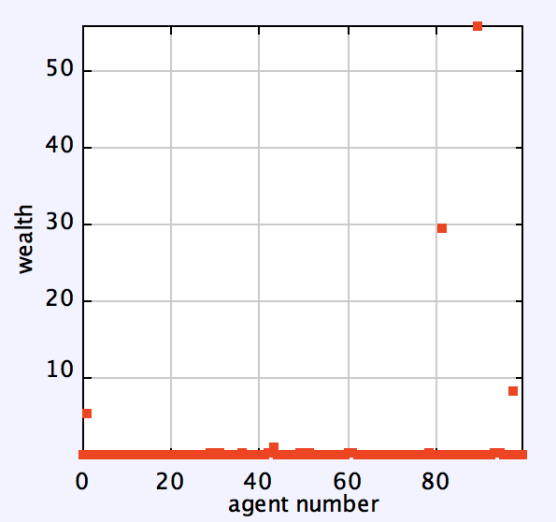


Figure 6: The wealth distribution after a million transactions. This simulation is available at <http://www.compadre.org/osp/EJSS/3901/model3/42.htm?F=1>.

Notice that as time goes on fewer and fewer agents have any significant wealth. After 100 transactions agent 79 has the most wealth, but after 10,000 transactions agent 32 has the most wealth. Then at 100,000 transactions agent 93 has taken over the top spot. After a million transactions it looks like agent 89 has taken over the top spot and now has over 50% of the entire wealth of the whole economy. If we ran the simulation long enough, eventually one agent will obtain essentially all the wealth.

Why does one agent obtain almost all of the wealth? The reason is that a percentage of the smaller wealth of the two agents is always transferred. Hence, richer agents lose a smaller percentage of their wealth compared to poorer agents. As most of the agents become poorer, richer agents rarely transfer wealth to other rich agents and hence rarely lose a significant percentage of their wealth. If we ran the model again we would again find that one agent ultimately receives almost all of the wealth, while a small number of agents typically jockey for the top spot until that agent wins and gets almost everything.

The model we have discussed is too simple to represent the actual trades that people, corporations, and other entities undergo every day. Economies are complex systems that have many influences which are difficult to include in a simple model. For example, you might object to using the same percentage of the poorer agent for every exchange. However, if we were to change the value of this percentage, we would still obtain the same result that almost all the wealth goes to one agent. Changing the value of the exchange percentage impacts only how long it will take for almost all the wealth to finally go to one agent. Changing the initial wealth and how it is distributed initially might bias which agent finally obtains almost all the wealth, but will not change the behavior of the model. That is, almost all of the wealth will go to one or a few agents in a short time, but eventually one agent obtains almost all the wealth. If we obtain the same behavior for different values of the initial wealth, number of agents, and the percentage of the poorer agents wealth to be transferred, we can have some confidence that the behavior of this simple model is able to capture the qualitative behavior of real economies.

More important is the insight that random exchanges such as those in the model drive the economy toward the accumulation of wealth. Thus, wealth accumulation is not an anomaly, but the rule.

5 How can we reduce wealth inequality?

We next consider some generalizations of the model to see what can be done to reduce the excessive wealth inequality that occurs. One possible way of avoiding wealth accumulation is to not allow

agents to lose a part of their wealth on most economic exchanges. If every good was priced exactly at its value and if every worker received in wages the added value of the products they produced, everybodys wealth would remain constant. However, this scenario is not realistic and not even desirable because profits can be used to grow the economy for the benefit of all. Instead, we need to look for other mechanisms that might lead to greater wealth equality.

One mechanism that might result in reducing wealth inequality is taxation. We will introduce three types of taxes in the model. One type taxes the amount gained in a transaction. This tax is similar to a sales tax or more accurately a tax on the profit made by one agent in each transaction. For simplicity, we will refer to this tax as a sales tax because this terminology is more familiar. Another approach is to tax the change in wealth over some interval of time; such a tax is similar to an income tax. Or we can tax the amount of wealth itself. The closest tax of this kind is a property tax. In addition to the different types of taxes, there may be different tax rates, depending on the amount of wealth or income. The rate for sales taxes usually is the same for all exchanges, except for food and clothing which are excluded in some states.

The following simulations that we discuss incorporate the effects of a flat sales tax for which the tax rate is the same regardless of the amount of the exchange, and progressive income and wealth taxes for which the tax rate increases with income and wealth. For simplicity, we set the income tax rate at 30% for income over $50,000, 20% for income between $20,000 and $50,000, and 10% for income between $10,000 and $20,000. These rates are roughly comparable to the income tax rate in the United States. The wealth tax rate in the model is 3% for wealth over $500,000, 2% for wealth between $200,000 and $500,000, and 1% for wealth between $100,000 and $200,000. For simplicity, we choose the sales tax rate to be 10%.

The way we tax is only half of the story. The other half is what we do with the revenue from taxes. Politicians and the media constantly debate the former, but seldom do they discuss how tax revenue is distributed. Part of tax revenue goes to running governments, such as salaries for government officials, costs to build and maintain government offices, and the military. Another large part is for subsidies of various economic interests such as agribusiness, oil companies, and transportation. Some tax revenue is used to fund social programs, but it is not common for tax revenue to specifically address wealth inequality.

We will introduce two ways of distributing tax revenue; reality is somewhere in between these two limiting ways. One way is to distribute the tax revenue so that the wealth of the agents is increased by the same percentage of their existing wealth. We will refer to this method of distribution as proportional revenue distribution, because wealthier agents receive more of the tax revenue than poorer agents. Another way is to give each agent an equal amount, regardless of the agents wealth. We will call this way equal revenue distribution. Equal tax revenue distribution is rarely done directly, but instead is in the form of social services such as free or inexpensive health care, education, child support, and a safety net for those who are disabled or unemployed.

The model collects the sales tax after every exchange, and income and wealth taxes after every N transactions. Tax revenue is distributed after N transactions. For simplicity, we start all agents with the same amount of wealth. Before you do the simulations, which taxation mechanism and revenue distribution do you think most benefits the poorer agents?

6 What have we learned?

Here is a summary of what happens when taxes are added.

Both the sales tax and the income tax lead to almost all the wealth eventually going to one agent if a proportional revenue distribution is used, just as if there was no tax at all. The main difference is that the rich get richer even faster because more of the tax dollars go to the rich.

A progressive wealth tax with a proportional revenue distribution leads to two groups of agents: one group (about 40% of the agents) has almost nothing, and the rest have some wealth. Here is what the wealth distribution looks like.

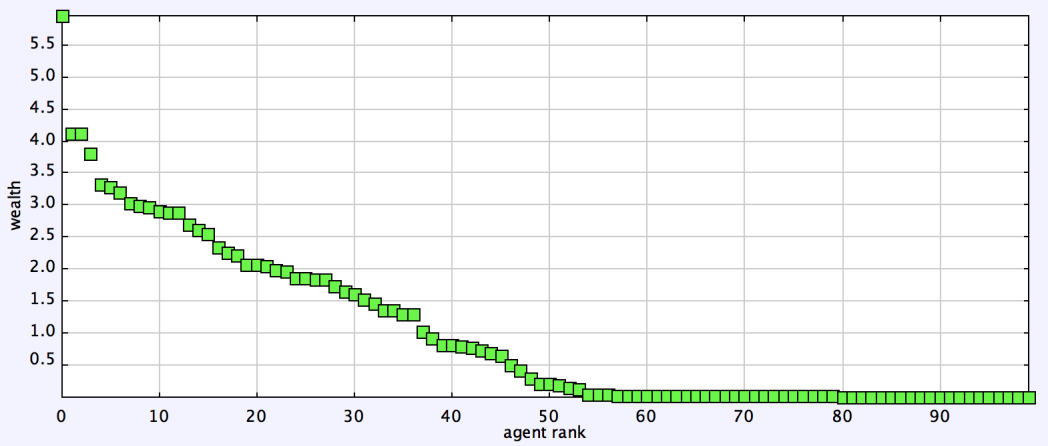


Figure 7: The wealth distribution with a progressive wealth tax and proportional revenue distribution. This simulation is available at <http://www.compadre.org/osp/EJSS/3901/model4/43.htm?F=1>.

Wealth inequality is reduced if equal revenue distribution is chosen. The poorest agent always has some wealth because every agent receives an equal amount of the tax revenue. For the income tax the wealthiest agent has about 500 times more wealth than the poorest; for the sales tax the ratio is about 300; and for a wealth tax it is only about 50. These ratios would change depending on the tax rates and how often the tax is collected and distributed. Here we show the wealth distributions for equal revenue distributions for the different kinds of taxes.

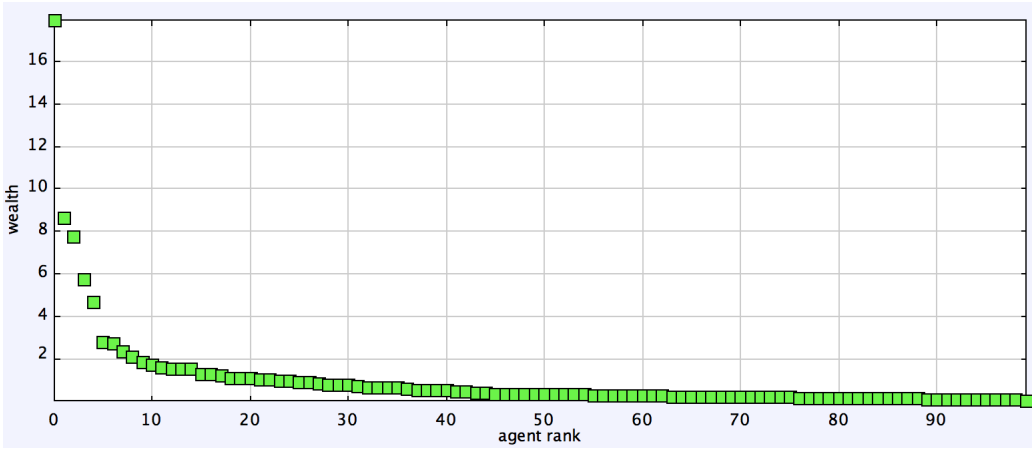


Figure 8: The wealth distribution with a 10% sales tax and equal revenue distribution. This simulation is available at <http://www.compadre.org/osp/EJSS/3901/model4/43.htm?F=1>.

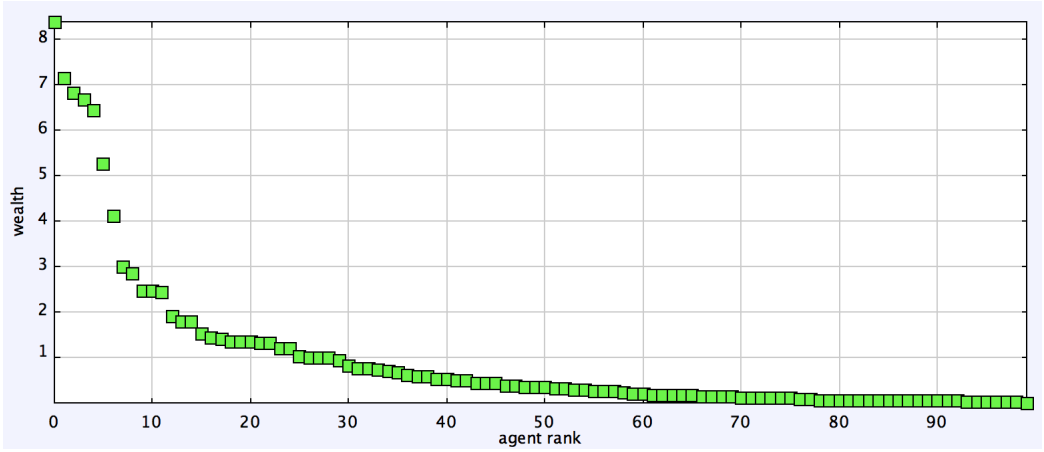


Figure 9: Wealth distribution with a progressive income tax and equal revenue distribution. This simulation is available at <http://www.compadre.org/osp/EJSS/3901/model4/43.htm?F=1>.

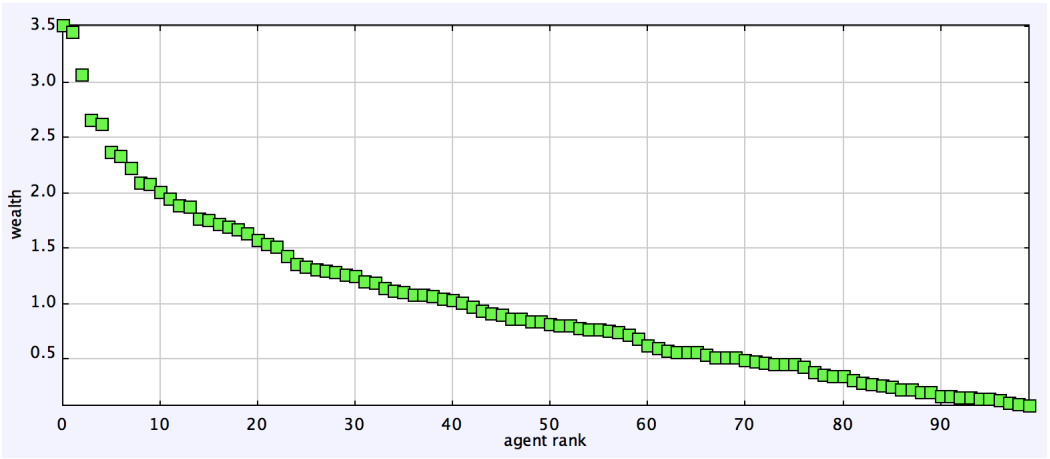


Figure 10: Wealth distribution with a progressive wealth tax and equal revenue distribution. This simulation is available at <http://www.compadre.org/osp/EJSS/3901/model4/43.htm?F=1>.

In the United States the wealthiest person has billions of times more wealth than the very poor, who have little or even negative wealth if they are in debt. Many people believe that the poor receive more from the government than the rich. However, the results of our simple model suggest the opposite conclusion, because we found that an equal distribution of wealth results in much more equality than exists in the United States. Consistent with this conclusion is the report by the Corporation for Enterprise Development that "...an analysis of $340 billion in tax subsidies for housing, education, retirement and savings in 2013, [showed that] the top 1 percent received about $95 billion, more than the $90 billion received by the bottom 80 percent combined." As noted, there are big subsidies to various industries such as agribusiness and oil and gas companies which do not directly benefit the lowest strata of society. The United States is a long way from using the tax system to promote wealth equality or even to prevent excessive wealth inequality.

7 Gini Coefficient: A measure of inequality

By looking at the distribution of wealth among the agents we were able to conclude that various forms of taxation and distribution can lead to greater or lesser wealth equality. However, it is

useful to introduce an explicit measure of wealth inequality so that we can reach more quantitative conclusions.

A common measure of wealth inequality is the Gini coefficient (or Gini index), G. A small value of G indicates a more equal wealth distribution with $G = 0$ corresponding to complete equality. In contrast, $G = 1$ corresponds to one person having all the wealth. The Gini coefficient for wealth varies from about 0.55 for Japan to 0.85 for Namibia, and is about 0.80 for the United States. Denmark, which has a strong welfare program, has a Gini coefficient of 0.81. The Gini coefficient for disposable income is typically lower, between 0.3 and 0.5, and is about 0.45 in the United States and 0.30 in Denmark.

The meaning of the Gini coefficient can be best understood by looking at Figures 11 and 12, which show plots of the percentage of the total wealth of a population owned by a given percentage of the population, starting from the poorest person. This percentage is an example of a cumulative distribution and is equal to the percentage of the population having wealth less than or equal to a given amount. As this amount is increased, the cumulative wealth distribution must also increase. For perfect wealth equality $G = 0$, and the cumulative wealth distribution grows linearly with the number of people as we see in the following plot.

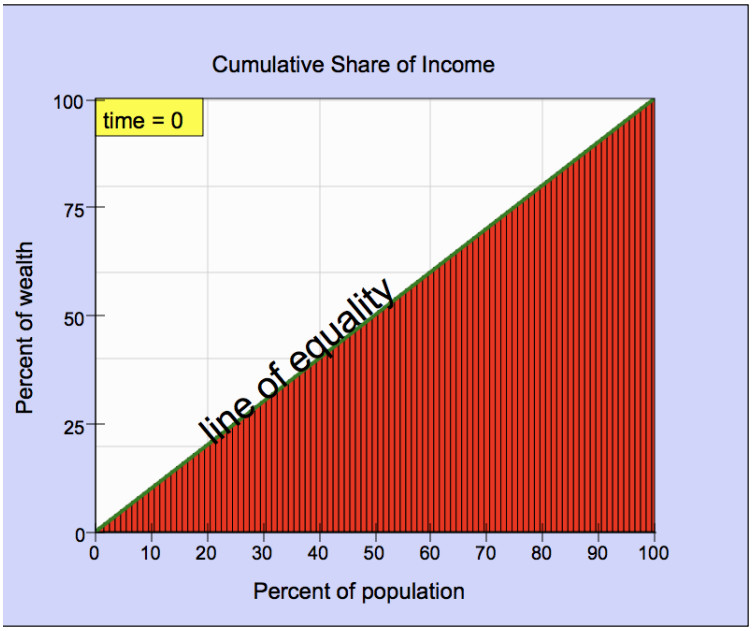


Figure 11: Illustration of the Gini coefficient for perfect equality. This simulation is available at `<http://www.compadre.org/osp/EJSS/3901/model5/44.htm?F=1>`.

If wealth is distributed unequally, we obtain a curve for the cumulative wealth instead of a straight line indicating that some people have more wealth than others. If the beginning of the curve is flat, that means many people have very little wealth. The end of the curve on the right must reach 100% of the total wealth in the economy. A very sharp rise near the right of the curve indicates that a few people have most of the wealth.

The Gini coefficient equals the area between the line of equality and the cumulative wealth of the agents divided by the area under the line of equality as shown in the figure below. This plot corresponds to a value of $G = 0.42$ when the tax revenue from a progressive wealth tax is distributed equally to all agents.

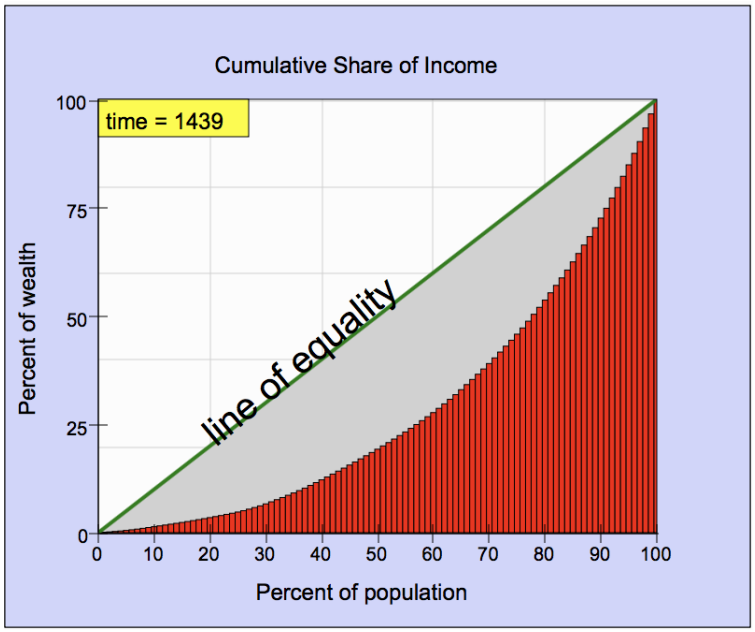


Figure 12: Illustration of the Gini coefficient equal to 0.42 found from a progressive wealth tax with equal revenue distribution. This simulation is available at <http://www.compadre.org/osp/EJSS/3901/model5/44.htm?F=1>.

The results of simulations using the models we have discussed show that for proportional distribution of tax revenue, the Gini coefficient is greater than 0.9 for income and sales taxes and about 0.65 for a wealth tax.

If revenue is distributed evenly, the smallest Gini coefficient is 0.42 for the wealth tax, 0.69 for the income tax, and 0.63 for the sales tax. Again reality is somewhere between equal and unequal tax revenue distribution, but closer to the value we obtain for proportional distributions for most countries.

8 Summary

We have learned by simulating several simple models that the result of random exchanges of wealth between agents causes the wealth to eventually accumulate in a single agent, leading to excessive wealth inequality. We also saw that it is important to have some type of wealth distribution to reduce wealth inequality. A progressive tax on wealth plus an equal distribution of tax revenue leads to the greatest reduction of wealth inequality among the various taxation and tax revenue distribution mechanisms that we considered.

The big question that is unanswered by the models that we have considered is what policies lead to greater economic growth per capita? Recent studies have shown that if the wealth inequality in a society is too high, economic growth is also limited. If there are too many poor people, who will buy the goods that are produced? So there is much work to be done to understand how to best balance the desire for a just society with a rising standard of living for everyone.

References

[1] "Income inequality in the United States," Wikipedia (accessed November 23, 2015).

[2] Chad Stone, Danilo Trisi, Arloc Sherman, and Brandon Debot, "A guide to statistics on historical trends in income inequality," Center on Budget and Policy Priorities (accessed November 23, 2015).

[3] Thomas Picketty, *Capital in The Twenty-First Century*, Harvard University Press (2013).

[4] The March 17, 2015 issue of the *New York Times* has an article that argues that tax revenue distribution benefits mostly the rich.

[5] The terms "income" and "wealth" are frequently used synonymously, but represent different quantities. Wealth includes all assets such as savings, real estate, and personal property. Income is due to wages, interest, dividends, etc. One advantage of considering income rather than wealth is that data on income is readily available. For example, the United States Internal Revenue Service has much data available in the form of a spreadsheet. Much historical information about income inequality in the United States can be obtained from census data.

[6] "Tax statistics," United States Internal Revenue Service (accessed December 1, 2015).

[7] "Historical income tables: Income inequality," United States Census Bureau (accessed December 1, 2015).

[8] The model we have discussed is an example of an agent-based model and has been mainly developed by physicists. It is commonly referred to as the asset exchange or yard sale model. Read some of the original papers to learn more about this model.

[9] Brian Hayes, "Follow the money," American Scientist **90** (5), 400–4005 (2002).

[10] Victor M. Yakovenko and J. Barkley Rosser, Jr., "Colloquium: Statistical mechanics of money, wealth, and income," Rev. Mod. Phys. **81**, 1703–1725 (2009).

[11] P. L. Krapivsky and S. Redner, "Wealth distributions in asset exchange models," Science and Culture **76**, 424–429 (2010).

[12] Anirban Chakraborti, Ioane Muni Toke, Marco Patriarca Toke, and Frédŕic Abergel, "Econophysics: Empirical facts and agent-based models," arXiv: 0909.19742-1–51 (2010).

[13] Anirban Chakraborti, Ioane Muni Toke, Marco Patriarca, and Frédéric Abergel, "Econophysics review: II. Agent-based models," Quantitative Finance **11**(7), 1013–1041 (2011).

[14] Bruce M. Boghosian, "Kinetics of wealth and the Pareto law," Phys. Rev. E **89**, 042804-1–22 (2014).